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PMM/KS/15/5832

Bachelor of Science (B.Sc.) Semester—IV

Examination

MATHEMATICS

Paper—II

(M₈-Mechanics)

Time—Three Hours]

[Maximum Marks—60

N.B.:—(1) Solve all the FIVE questions.

(2) All questions carry equal marks.

(3) Questions 1 to 4 have alternatives. Solve each question in full or its alternative in full.

UNIT—I

1. (A) If six forces of relative magnitudes 1, 2, 3, 4, 5 and 6 act along the sides of a regular hexagon taken in order, show that the single equivalent force is of relative magnitude 6 and that it acts along a line parallel to the force 5 at a distance from the centre of the hexagon $3\frac{1}{2}$ times the distance of a side from the centre. 6

- (B) Five weightless rods of equal length are joined together so as to form a rhombus ABCD with one diagonal BD. If a weight W be attached to C and the system be suspended from A, show that there is a thrust in BD equal to $W/\sqrt{3}$. 6

OR

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Contd.

- (C) A given length '2s' of a uniform chain has to be hung between two points at the same level and the tension has not to exceed the weight of a length 'b' of the chain. Show that the greatest span is :

$$\sqrt{(b^2 - s^2)} \log[(b+s)/(b-s)]. \quad 6$$

- (D) If α and β be the inclinations to the horizon of the tangents at the extremities of a portion of a common catenary, which are on one side of the vertex of the catenary, and 'l' the length of the portion, show that the height of one extremity above the other is :

$$l \sin\left(\frac{\alpha + \beta}{2}\right) / \cos\left(\frac{\alpha - \beta}{2}\right). \quad 6$$

UNIT—II

2. (A) The velocities of a particle along and perpendicular to a radius vector from a fixed origin are λr^2 and $\mu \theta^2$. Show that the equation to the path is

$$\frac{\lambda}{\theta} = \frac{\mu}{2r} + c \text{ and the components of accelerations are}$$

$$2\lambda^2 r^3 - \mu^2 \frac{\theta^4}{r} \text{ and } \lambda \mu r \theta^2 + 2\mu^2 \frac{\theta^3}{r}. \quad 6$$

- (B) An insect crawls at a constant rate 'u' along the spoke of a cart wheel of radius 'a', the cart is moving with velocity 'v'. Find the accelerations along and perpendicular to the spoke. 6

OR

- (C) A point moves in a plane curve, so that its tangential and normal accelerations are equal and the angular velocity of the tangent is constant. Find the curve.

6

- (D) At the ends of three successive seconds the distances of a point moving with simple harmonic motion from its mean position measured in the same direction are 1, 5 and 5. Show that the period of a complete oscillation is $\frac{2\pi}{\cos^{-1}(3/5)}$.

6

UNIT—III

3. (A) Show that Lagrange's equations :

$$\frac{d}{dt} \left[\frac{\partial T}{\partial \dot{q}_j} \right] - \frac{\partial T}{\partial q_j} = Q_j \text{ can also be}$$

$$\text{written as } \frac{\partial \dot{T}}{\partial \dot{q}_j} - 2 \frac{\partial T}{\partial q_j} = Q_j, \quad j=1, 2, \dots, n,$$

where $Q_j \equiv$ the generalized forces

$T \equiv$ the kinetic energy of the system

$q_j \equiv$ the generalized coordinates

$\dot{q}_j \equiv$ the generalized velocities.

6,

- (B) Let M_1 and M_2 be the masses of two particles with $M_1 > M_2$, connected by the light rope which passes over the pulley, assumed to be frictionless and massless, then from the equation of motion, show that :

$$\ddot{x} = \left[\frac{M_1 - M_2}{M_1 + M_2} \right] g, \text{ where } \ddot{x} = \text{the common}$$

acceleration of the particle and g = gravitational acceleration. 6

OR

- (C) Prove that $2F$ is the rate of energy dissipation due to friction, where F is Rayleigh's dissipation function. 6

- (D) If L is a Lagrangian for a system of 'n' degrees of freedom satisfying Lagrange's equations, show by direct substitution that $L' = L + \frac{dF}{dt}(q_1, \dots, q_n, t)$, also satisfies Lagrange's equations, where F is any arbitrary, but differentiable, function of its arguments. 6

UNIT—IV

4. (A) Prove that the central force motion of two bodies about their center of mass can always be reduced to an equivalent one body problem. 6

- (B) If V is a power-law function of r such that $V = ar^{n+1}$, then prove that the virial theorem takes a form $\bar{T} = -\frac{1}{2}\bar{V}$. 6

OR

- (C) In a central force field, prove that the relation between 'r' and 't' is given by :

$$t = \int_0^r \frac{dr}{\sqrt{\frac{2}{m} \left[E - V - \frac{\ell^2}{2mr^2} \right]}}$$

where E = total energy, V = potential energy. 6

- (D) For a general system of mass points with position vector $\vec{r}_i$ and applied forces $\vec{F}_i$, including any forces of constraint, prove that $\bar{T} = -\frac{1}{2} \sum_i \overline{\vec{F}_i \cdot \vec{r}_i}$, where T = kinetic energy of the system. 6

Question—V

5. (A) State the principle of virtual work for a system of coplanar forces acting on a particle. 1½
- (B) For a common catenary, show that $s = c \sinh(x/c)$. 1½
- (C) If the radial and transverse velocities of a particle are proportional to each other, then show that the path is an equiangular spiral. 1½



(D) If tangential and normal acceleration components of a particle are equal, prove that its velocity varies as e^x . 1½

(E) If the total force $\vec{F}$ is zero then prove that $\dot{\vec{p}} = 0$ and linear momentum $\vec{p}$ is conserved. 1½

(F) State D'Alembert's principle. 1½

(G) In a central force field, prove that the path of a particle lies in a plane. 1½

(H) Define the central force and the central orbit. 1½

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